

Induced Magnetic Fields

We know that a changing magnetic flux induces an electric field, and we ended up with Faraday's law of induction in the form

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt} \quad (\text{Faraday's law of induction}).$$

Here \vec{E} is the electric field induced along a closed loop by the changing magnetic flux Φ_B encircled by that loop. Because symmetry is often so powerful in physics, we should be tempted to ask whether induction can occur in the opposite sense; that is, can a changing electric flux induce a magnetic field?

The answer is that it can; furthermore, the equation governing the induction of a magnetic field is almost symmetric with above equation. We often call it Maxwell's law of induction after James Clerk Maxwell, and we write it as

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt} \quad (\text{Maxwell's law of induction}).$$

where i_{enc} is the current encircled by the closed loop. Thus, our two equations that specify the magnetic field produced by means other than a magnetic material (that is, by a current and by a changing electric field) give the field in exactly the same form. We can combine the two equations into the single equation

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{\text{enc}} \quad (\text{Ampere-Maxwell law}).$$

When there is a current but no change in electric flux (such as with a wire carrying a constant current), the first term on the right side of above equation is zero, and so it reduces to.

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{enc}} \quad (\text{Ampere's law}).$$

When there is a change in electric flux but no current (such as inside or outside the gap of a charging capacitor), the second term on the right side of Ampere-Maxwell law equation is zero, and so it reduces to Maxwell's law of induction equation.

The Basic Equations of Electromagnetism

Maxwell's equations

Equation of Ampere-Maxwell law is the last of the four fundamental equations of electromagnetism, called *Maxwell's equations* and displayed in Table below. These four equations

Name	Equation	
Gauss' law for electricity	$\oint \vec{E} \cdot d\vec{A} = q_{\text{enc}}/\epsilon_0$	Relates net electric flux to net enclosed electric charge
Gauss' law for magnetism	$\oint \vec{B} \cdot d\vec{A} = 0$	Relates net magnetic flux to net enclosed magnetic charge
Faraday's law	$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$	Relates induced electric field to changing magnetic flux
Ampere–Maxwell law	$\oint \vec{B} \cdot d\vec{s} = \mu_0\epsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{\text{enc}}$	Relates induced magnetic field to changing electric flux and to current

explain a diverse range of phenomena, from why a compass needle points north to why a car starts when you turn the ignition key. They are the basis for the functioning of such electromagnetic devices as electric motors, television transmitters and receivers, telephones, scanners, radar, and microwave ovens.